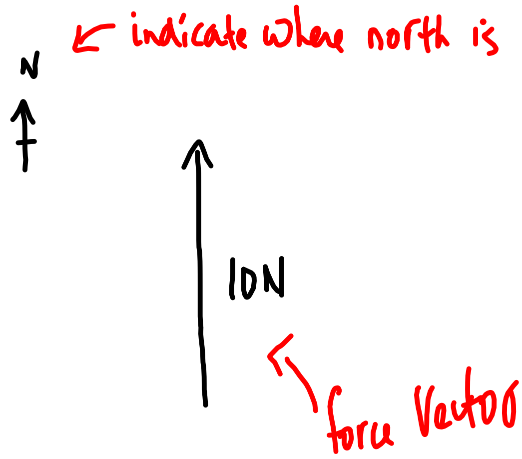
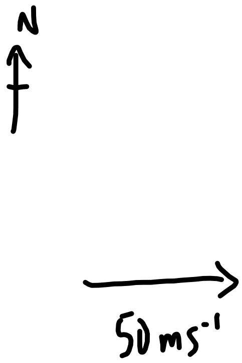
Representation of Vectors

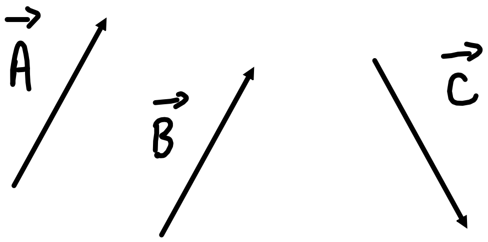
$\vec{F} = 10N$ north
[N]



$\vec{V} = 50\,ms^{-1}$ east
[E]



Equality of Vectors



$\vec{A} = \vec{B}$

(same size + same direction)

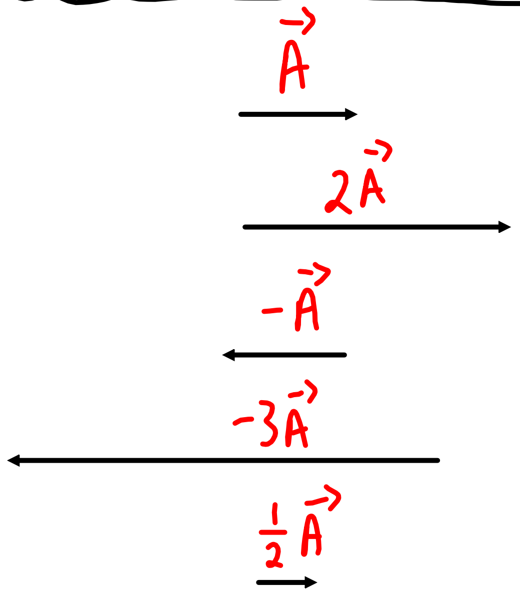
$\vec{A} \neq \vec{C}$

(same size but diff. direction)

$|\vec{A}| = |\vec{C}|$

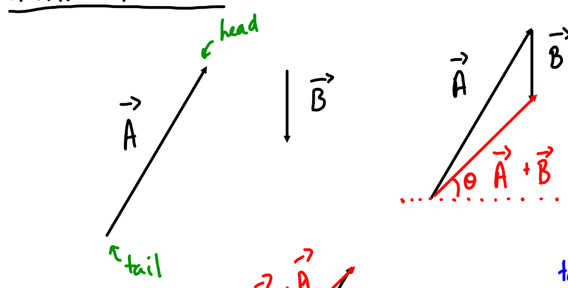
lines mean "magnitude".

Multiplication of a Vector by a Scalar:



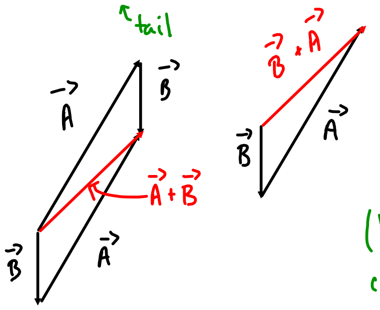
- Direction stays the same unless multiplied by a negative scalar
- magnitude changes.

Addition of Vectors



join the vectors head to tail

(resultant tail of the first to head of the second vector)



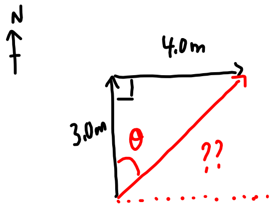
$\vec{B}+\vec{A}=\vec{A}+\vec{B}$

(Vector addition is commutative)

(i.e. the order does not matter)

Example

Find the sum of two displacement vectors 3.0m north and 4.0m east.



$c^2 = a^2 + b^2$

$c^2 = (3.0m)^2 + (4.0m)^2$

$c = 5.0m$ ← only the magnitude.

Direction: ~~SOH~~ ~~CAH~~ TOA

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{4.0m}{3.0m}$

$\theta = \tan^{-1}\left(\frac{4.0m}{3.0m}\right)$

$\theta \approx 53^\circ$ ← direction

The displacement is 5.0m [N53°E]

5.0m [E 37° N]

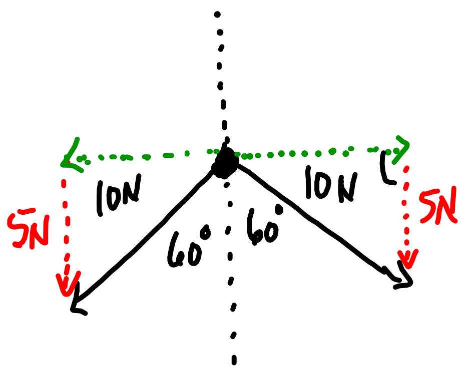
5.0m 53°T

↑ a true bearing of 53°

(For bearings, angles are measured CW)

Example

Two forces, each of magnitude 10N, act on a nail. One force is inclined downwards at 60° to the left of vertical and the other is inclined downwards at 60° to the right of vertical. What is the resultant force acting on the nail?



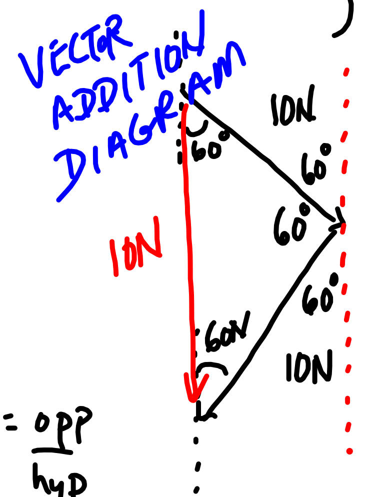
FBD (Free Body Diagrams)

(Note: you cannot add vectors here since not head to tail)

$\sin\theta = \frac{opp}{hyp}$

$\sin 30° = \frac{x}{10}$

$x = 5$ N

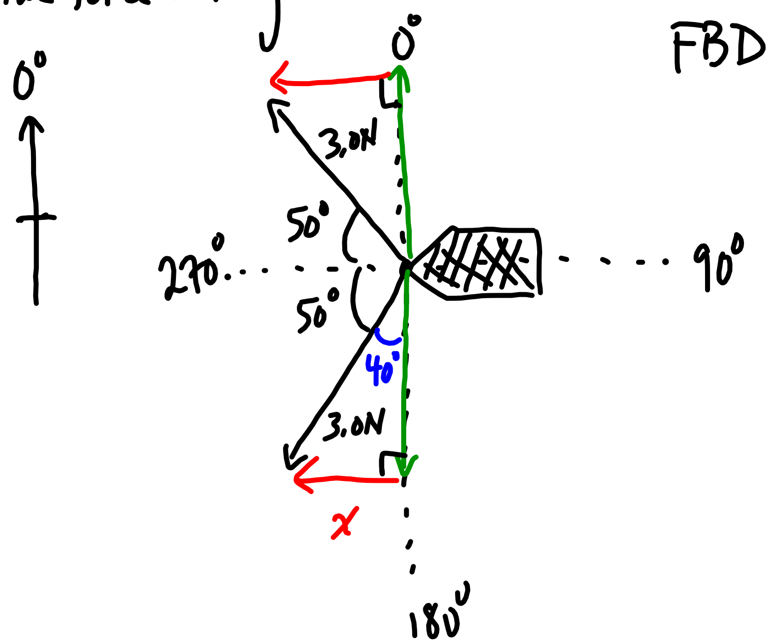


an equilateral triangle.

The resultant force is 10N [downwards]

Example

Two forces, each of magnitude 3.0 N, act on the front of a toy boat. One of the forces acts in a direction of 320°T and the other in a direction of 220°T. Determine the total force acting on the boat.



$$\sin\theta = \frac{opp}{hyp}$$

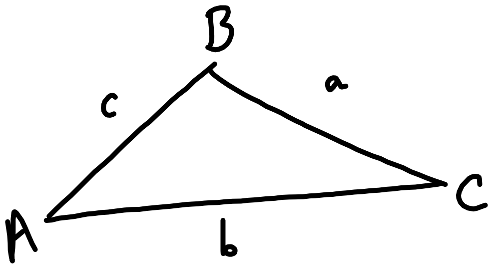
$$\sin 40° = \frac{x}{3.0N}$$

$$x = (3.0N)\sin 40°$$

$$x = 1.9\ N$$

So the resultant force is $2(1.9N) = 3.9N$
270°T

Non-Right Triangles



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

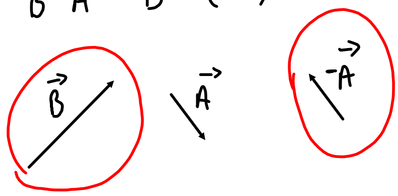
Law of Cosines

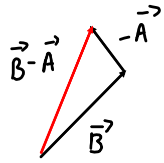
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Subtraction of Vectors

$5 - 3 = 5 + (-3)$

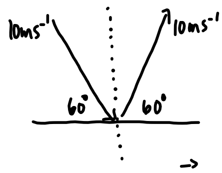
$\vec{B} - \vec{A} = \vec{B} + (-\vec{A})$





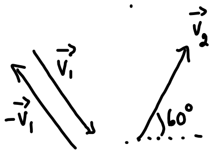
Example

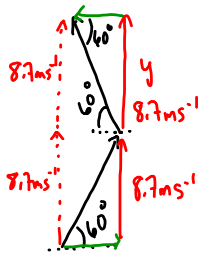
A billiard ball moving with a velocity of $10\,ms^{-1}$ inclined at $60°$ to the edge of the table bounces off the edge of the table at the same angle but with no change in speed. Determine the change in velocity of the ball.



$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$\Delta\vec{v} = \vec{v}_2 + (-\vec{v}_1)$





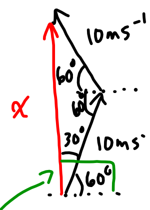
$\sin\theta = \frac{opp}{hyp}$

$\sin 60° = \frac{y}{10\,ms^{-1}}$

$y = (10\,ms^{-1})\sin 60°$

$y = 8.7\,ms^{-1}$

$\Delta\vec{v} = 17\,ms^{-1}$ directly away from edge.



because of symmetry this angle is $30°$ (straight away from edge)

Law of Cosines. or

Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b}$

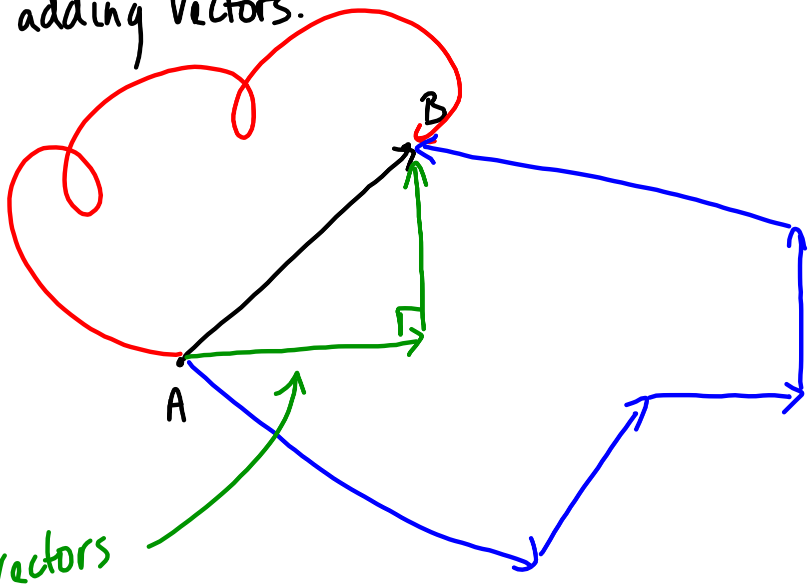
$\frac{\sin 30°}{10\,ms^{-1}} = \frac{\sin 120°}{x}$

$x = \frac{(\sin 120°)(10\,ms^{-1})}{\sin 30°}$

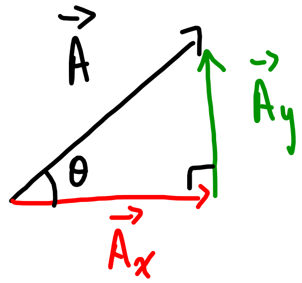
$\boxed{x = 17\,ms^{-1}}$

Components of Vectors

Resolving a vector into components is really just the opposite of adding vectors.



These vectors are components
(the *must* be perpendicular to one another)



(vertical component of $\vec{A}$)

(horizontal component of $\vec{A}$)

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin\theta = \frac{A_y}{A}$$

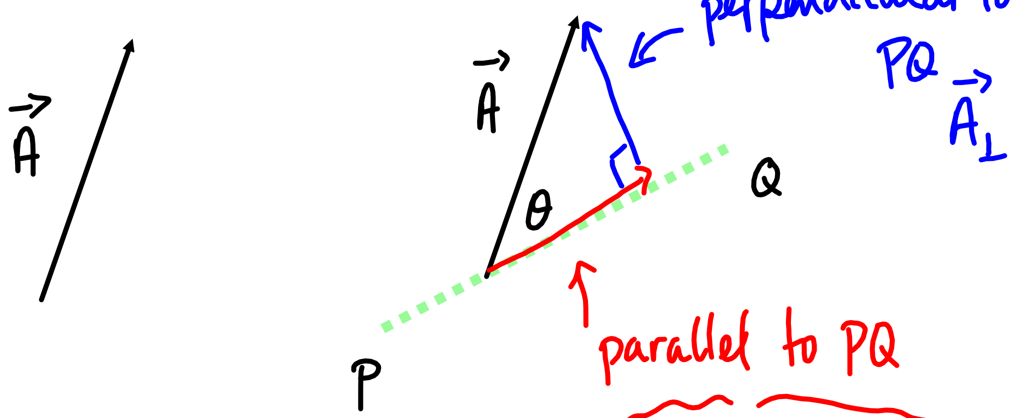
$$\cos\theta = \frac{adj}{hyp}$$

$$\cos\theta = \frac{A_x}{A}$$

$$\tan\theta = \frac{A_y}{A_x} \quad \frac{(opp)}{(adj)}$$

Resolving a vector into parallel and perpendicular components in relation to direction PQ.

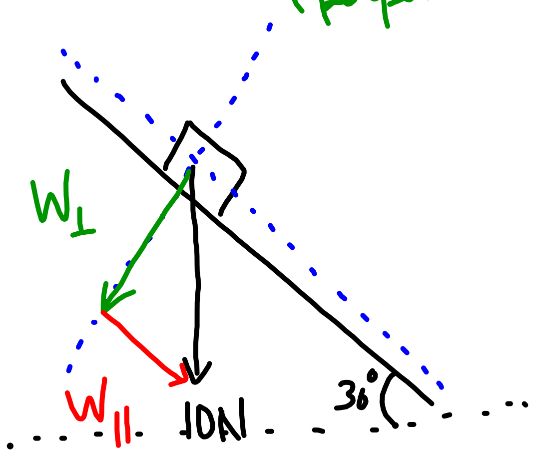
$A_\perp = A\sin\theta$



$A_\parallel = A\cos\theta$

Example

A 10N weight is placed on a board which is inclined at an angle of 30° to the horizontal. Determine the components of the weight acting down (parallel to incline) the incline and normal to the incline. (perpendicular)



$W_\perp = W\cos\theta$

$W_\perp = 10N\cos 30^\circ$

$W_\perp = (10N)\left(\frac{\sqrt{3}}{2}\right) = (5\sqrt{3})N$

$W_\parallel = W\cos\theta$

$W_\parallel = 10N\sin 30^\circ$

$W_\parallel = (10N)\left(\frac{1}{2}\right) = 5N$